APPENDIX D

COMPUTATIONAL FORMULAE AND CONSTANTS

D.1 <u>Radiosonde Sensors and Data Reduction</u>. Pressure - the conversion from the engineering units to meteorological units will be specific to the radiosonde and its manufacturer. Values ***shall*** be converted to and reported in hectoPascals (hPa). The hectoPascal is equivalent to the previously used millibar.

Temperature - as with pressure, the conversion will be specific to the radiosonde. Temperatures ***shall*** be recorded in degrees Kelvin or degrees Celsius.

Relative Humidity - the same applies. Relative Humidity ***shall*** be recorded in per cent.

Wind - NAVAID-derived winds will be under the control of the manufacturer and may not be the same for the same radiosonde type. Wind information ***shall*** be recorded in polar form, in degrees from true north and in knots. A secondary form may be in vector form and in meters per second. Winds ***shall*** be computed using earth spherical geometry, insuring that accurate determinations result in the cases of high mean wind.

D.2 <u>Geopotential Height</u>. The altitude of the radiosonde is reported in units of scaled geopotential (geopotential height) above mean sea level (MSL). The elevation of the release point (typically the station elevation) ***should*** be a surveyed geometric position, and ***shall*** be converted to geopotential height. The strata or layers (in pressure) resulting from the radiosonde telemetry rate ***shall*** be employed to calculate the geopotential height of the instrument. The time increments are usually the points sampled, but the values of the variables ***should*** be the result of the signal processing algorithm (Chapter 2) in order to achieve a balance between vertical resolution and the quality of the variables. Layer thicknesses ***shall*** be calculated from the measured pressures at the bounds of each layer and the mean virtual temperature within the layer.

The definition of geopotential is the potential energy due to gravity of a unit mass of air at some point above a standard position (i.e. zero energy), usually mean sea-level, and is measured in a positive sense vertically upward. Geopotential is:

$$\Phi = \int_0^Z g dZ \ .$$

The geopotential meter (gpm) is defined as a rescaled geopotential, and is given by:

$$h_n = \frac{1}{9.80665} \int_0^n g(z) dz \ .$$

Thus, a discrete version of the hypsometric equation is

$$\ln \frac{p_{i+1}}{p_i} = -\frac{g\,\delta h}{R^{*}Tv}$$

where;

p = pressure in hPa
g = acceleration of gravity (standard) = 9.80665 according to WMO Technical Regulations
R* = gas constant for dry air = 287.04 $m^2 s^{-2} K^{-1}$
Tv = mean virtual temperature for the layer = [Tv(i+1) + Tv(i)]/2 in degrees K
*h = layer thickness (meters)
i refers to lower boundary.

For simplicity, the equation above can also be expressed as:

$$\Delta h_i = -14.6355\,(T_{i-1} + T_i)\ln(\frac{p_{i-1}}{p_i})$$

Finally, the thickness) h_i of each pressure layer above the surface in geopotential meters may be summed to give the height h_n of any pressure surface. Thus,

$$h_n = \sum_{i}^{n} \Delta\, h_i$$

Occasionally it is necessary to convert from geopotential to geometric height. The conversion algorithm is:

$$z_n = \frac{h_n R_e}{Gr - h_n}$$

where

Gr = the gravity ratio = [g_N Re/9.80665]
Re = radius of the earth at latitude N , and
g_N = gravity at N = 9.80616[1-0.002637(cos(2N)) + 0.0000059($\cos^2$(2N))]

For constructing the Standard Atmosphere pressure-altitude (pressure-height) relationship, the following parameters are defined; they are identical for the ICAO and U.S. Standard Atmospheres (Ref. 15):

The pressure at zero altitude is 1013.250 hPa.
The Temperature at zero pressure-altitude is 288.16°K.
The lapse rate of temperature to 11.0km is -6.5°C per kilometer.
The lapse rate of temperature from 11.0 to 25km is 0.0°C per kilometer.
The lapse rate of temperature from 25.0 to 47km is +3.0°C per kilometer.

In using the hypsometric equation to construct the pressure-altitude relation, the following constants shall be used:

The acceleration of gravity is 9.80665 meters per second.
The gas constant R=R* (the atmosphere is assumed to be dry) is 287.04 $m^2s^{-2}K^{-1}$.

D.3 <u>Thermodynamic Relationships</u>. The basic moisture variable sensed by a radiosonde is the relative humidity. Other variables are calculated from this and the sensed air temperature. Relative humidity ***shall*** be defined as the ratio of the existing vapor pressure, e, to that at saturation, e_s, at temperature, T. If expressed in percent, the notation U is used; if in fractional form, u, thus

$$u = e/e_s \, .$$

The following expressions ***shall*** be used:

D.3.1 Vapor Pressure. The pressure due to the presence of water substance in gaseous form is given by

$$e_s = 6.1121 \exp[(17.502\,T)\,(240.97 + T)^{-1}]$$

where e_s = saturation vapor pressure in hectoPascals at temperature, T, in degrees Celsius. This approximation is valid for vapor pressure over water to -50° Celsius.

D.3.2 Virtual Temperature. Virtual temperature is derived from temperature and relative humidity, thus:

$$T_v = T_p[p - (0.37821\,e)]^{-1}$$

where, e = vapor pressure , a function of temperature only
p = measured ambient air pressure.

D.3.3 Dew Point and Dew Point Depression. The dew point temperature in degrees Celsius is approximated by

$$T_d = \frac{b(b+T)\ln u + aT}{ab - (b+T)\ln u}$$

where u is the fractional relative humidity at temperature T (degrees Celsius) and constants a = 17.502 and b = 240.97. This expression is consistent with D3, and has the advantage that u can be easily recovered if desired. The dew point depression is $T - T_d$, a positive definite quantity.

D.4 Mixing Ratio. Mixing ratio, r, in parts per thousand, is derived from the measured air pressure and the vapor pressure, thus

$$r = 0.622[e/(p-e)] \ .$$

D.5 Potential Temperature. The potential temperature is useful in checking and characterizing super-adiabatic lapse rates. It is defined as a reference temperature obtained by changing an air parcel's temperature adiabatically to a pressure of 1000hPa. Thus:

$$\theta = T\left(\frac{1000}{P}\right)^{\frac{R^*}{C_p}}$$

where R* is the gas constant for air and c_p is the specific heat of air at constant pressure. The quantity R^*/c_p is equal to 2/7 exactly.

D.6 Wind Variables. The calculation of wind from the position of the balloon ***shall*** be made using earth spherical coordinates. The position of the balloon ***should*** also be used for establishing the position of the information at the levels selected for the rawinsonde message (Chapter 5).

D.6.1 Wind Speed and Direction. Meteorological convention specifies wind direction as that FROM which the wind blows. A 360 degree compass oriented clockwise from north is used; thus, a wind blowing from the west is given a 270 degree direction, and from the south a 180 degree direction. This convention is in contra-distinction to Oceanographic convention, in which currents are labelled with the direction TOWARD which the current is flowing.

In meteorological usage the Cartesian velocity component values are calculated from the wind speed and direction. The u-component (east-west) of the wind is given by

$$u = \text{speed}[- \sin (\text{direction})]$$

and the v- component (north-south) by

$$v = \text{speed}[- \cos(\text{direction})].$$

In the inverse case, correct determination of the proper quadrant for the wind direction can be obtained by using the ATAN2 subroutine available in system software, or

if u = 0 and v < 0, direction = 360°
if u = 0 and v > 0, direction = 180°, otherwise

$$D_c = 57.29578 \cdot [\arctan(\frac{v}{u})]$$

and, if u < 0 then Direction = 270 - D_c,
if u > 0 then Direction = 90 - D_c.

Wind speed can be determined from position components by

$$Speed = (u^2 + v^2)^{\frac{1}{2}}.$$

D.6.2 Wind Shear. The magnitude of the wind shear between any level and the level immediately below it is given by:

$$Wind\ shear = [(u - u_{-1})^2 + (v - v_{-1})^2]^{\frac{1}{2}} \cdot (\Delta z)^{-1}.$$

D.6.3 Mean Wind. The mean wind (5.3.7) included in the coded message is calculated by averaging separately the u- and v-components obtained from the high-data-rate and time-tagged data file between the surface and the 1525m (5000 ft) level and between the 1525m and the 2048m (10,000 ft) levels. D.6.1 is used to convert to direction and speed.

D.7 <u>Location of the Balloon</u>. Earth spherical geometry ***shall*** be used for the location of the balloon relative to the launch site. When the height of the balloon has been calculated from integration of the hypsometric equation (Section D.2) independently from the measurement of the elevation angle of the ground tracking antenna (e.g. RDF systems):

$$d(km) = 6371(\frac{\pi}{2} - \beta - \sin^{-1}\left\{\frac{R_e}{(R_e + h)}\cos(\beta)\right\})$$

where d is the distance from the launch point, $ is the measured elevation angle (rad), Re is the radius of the earth (km), and h the height of the balloon (km).

D.8 <u>Three-Dimensional Balloon Location</u>. In the event that the radiosonde does not carry a pressure sensor and that the height of the balloon is established by NAVAID positioning, the balloon position in two dimensions can be obtained from the system itself.

D.9 <u>Logarithmic Pressure Ratio (for Interpolation to Selected Pressure)</u>. The logarithmic pressure ratio is used to interpolate pressure between levels:

$$P_r = \frac{\log(P_i) - \log(P_s)}{\log(P_i) - \log(P_{i+1})}$$

where, P_r = pressure ratio,
P_i = pressure of the lower bounding level,
P_{i+1} = pressure of the upper bounding level, and
P_s = pressure of the constant pressure surface being selected.

Interpolation of any parameter (height, temperature, relative humidity, etc) is carried out using:

$$X_{im} = P_r \cdot (X_{i+1} - X_i) + X_i$$

where, X_{im} = parameter value at interpolated level,
P_r = pressure ratio,
X_i = parameter value at lower bounding level, and
X_{i+1} = parameter value of the upper bounding level.